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OPTIMUM DESIGN OF DEFLECTORS FOR PELTON WATER TURBINES

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Abstract: *The paper herewith presents a methodology for calculation of deflectors for Pelton water turbines. The optimum values of the rotation-axis coordinates of the deflectors are determined by modification through a scheduled numerical experiment, so that a minimal torque, necessary for the operation of the deflector, could be achieved. A computation program for dimensioning of deflectors has been developed. Comparison with results, yielded by an experimental study of deflectors has been provided as well.*

1. INTRODUCTION

The deflector is a key element of the Pelton turbine construction. One of the most important requirements in the process of deflector design is the provision of a minimal value of torque, necessary for setting deflectors in motion. The latter is usually achieved through modification either of the position of the rotation axis or of the form of stream-lined surface [1]. A set of methods, which demand the execution of a significant number of routine calculations are implemented to this end, without however any guarantee whatsoever given as to the arrival at an optimum solution. Determining the momentum of the effect of the stream on the deflector is also required in view of dimensioning the drive system and the turbine governor.

The paper herewith presents a methodology for calculation of deflectors for Pelton water turbines, which overcomes the flaws of the traditional methods and achieves automation of design calculations; determining the optimum geometric parameters is accomplished through a scheduled numerical experiment.

2. CALCULATION DIAGRAM

The description of the algorithm, implemented in the design of Pelton turbine deflectors, is as follows:

a). Input data for calculation of the scheduled values for flow rate Q and head H, the diameter of the outlet of nozzle d_e, and the number of nozzles z_n.

b). An array of variants is being developed - their number, in accordance with the

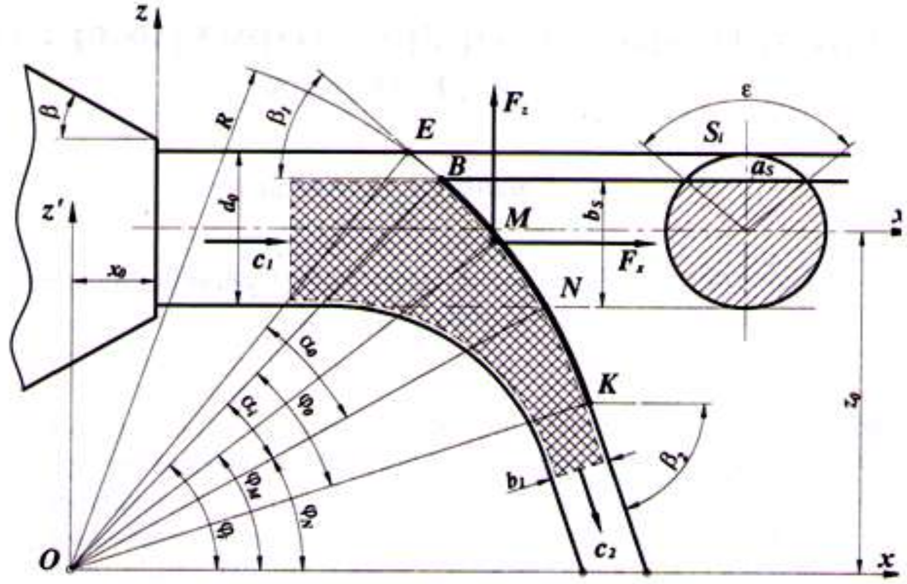

Fig. 1. Basic parameters of the jet and the deflector

implemented for the particular case D-optimum Kono [2] plan, equals the number of the points of carrier of the scheme $n = 9$. Operating parameters are the coordinates of the deflector's rotation-axis $x_0(x_1)$ и $z_0(x_2)$ - Fig.1. The values of the basic levels and the variation intervals of those parameters are determined on the basis of data for operating turbines, recommendations in specialized literature as well as a specialized numerical study performed exclusively to this end (Table 1).

Table 1

Factor	Basic level	Variation interval
x_1, m	$-0.2d_0$	$0.3d_0$
x_2, m	$z_{\min} + d_0/2$	$d_0/2$

The minimum value of the z_0 - coordinate could be determined in the following manner:

$$z_{\min} = |x_0| / tg\beta + d_0/2, \qquad (1)$$

where β stands for the nozzle angle; d_0 - diameter of the jet (Fig.1).

c). The value of the momentum of the hydrodynamic-power effect (the so-called 'hydraulic momentum' M_x) is determined at the scheme points for various positions (Fig.1) of the deflector $M_x = f(\alpha_i)$. Subsequently, the maximum value of the momentum M_{xm} is determined for every considered variant. The mathematical relationship of the latter factor (of the two) - the so-called 'regression model' - is of second degree and has the following mode:

$$y = b_0 + \sum_{i=1}^{2} b_i x_i + \sum_{1 \le i \le j \le 2}^{2} b_{ij} x_i x_j + \sum_{ii=1}^{2} b_{ii} x_{ii}^2 \tag{2}$$

The next stage consists of an assessment of the magnitude of the regression coefficients, and of a check for adequacy of the model [2].

d) The optimization problem is solved through the so-called complex method of M.Box, which is suitable in that particular case on account of the non-linearity of the model, and of the availability of limitations in the factor space. The total minimum of the target function y is determined after calculations at various starting points, and after comparison of the yielded values.

e) Dimensioning of the deflectors is accomplished through certain optimum (at $y \to \min$) values of the operating parameters x_1 и x_2.

3. DETERMINING THE MAIN GEOMETRIC PARAMETERS

Provided that the stream-lined surface of the deflector has a cylindrical form (Fig.1), and in view of arriving at a minimum value of the torque, the radius of that cylindrical surface is determined through the following formulae:

$$R = k(z_0 + d_e / 2) \quad \text{at } x_0 > 0, \tag{3}$$

$$R = k\sqrt{x_0^2 + (z_0 + d_e / 2)^2} \quad \text{at } x_0 < 0. \tag{4}$$

The coefficient k ($k = 1.05 \div 1.1$) guarantees the application of the technological requirement that the deflector at its endmost position be outside the opening of the nozzle.

Determining the main geometric parameters of the deflector, in case of known rotation-axis coordinates and cylindrical form of the stream-lined surface, is accomplished through the following expressions:

$$\varphi_N = \arcsin\frac{z_0 - d_e / 2}{R}, \tag{5}$$

$$\alpha_0 = \arcsin\frac{z_0 + d_e / 2}{R} - \varphi_N, \tag{6}$$

$$\varphi_0 = k_1\alpha_0 \ , \tag{7}$$

$$\alpha_i = \varphi_i - \varphi_N \ , \tag{8}$$

$$\beta_2 = \varphi_0 - \frac{\pi}{2} - \varphi_i \ , \tag{9}$$

where:

φ_N - angle, determining the position of the deflector before its incising into the jet, when the latter has a maximum diameter;

α_0 - angle, determining the so-called active arc;

φ_0 - angle of the range of the deflector $(k_1 = 1.1)$;

α_1 - angle, determining the degree of incision of the deflector into the jet;

β_2 - angle at the outlet of the deflector;

4. DETERMINING THE HYDRAULIC MOMENTUM

For the present purposes is used the theorem for kinetic moment (the control surface is shown on Fig. 2.)

$$\overrightarrow{M}_x = \rho Q_i (\overrightarrow{c_1} \times \overrightarrow{r_1} - \overrightarrow{c_2} \times \overrightarrow{r_2}) \ . \tag{10}$$

It is convenient for the momentum M_x to be determined through the projection of the hydrodynamic force on the x и z axes, i.e. F_x и F_z:

$$M_x = F_x z - F_z x \ , \tag{11}$$

where x and z are coordinates of the application point M of the forces (Fig.1).

$$F_x = \rho Q_i (c_1 - c_2 \cos\beta_2) \ , \tag{12}$$

$$F_z = \rho Q_i c_2 \sin\beta_2) \ , \tag{13}$$

$$z = z_0 - d_0/2 + b_s/2 \ , \tag{14}$$

$$x = z \sin\varphi_M \ , \tag{15}$$

where:

Q_i - flow, deflected from the jet; $Q_i = Q_d(1 - S_i / S_0)$;

Q_d - flow through nozzle;

S_0, S_i - areas, respectively of the section of the jet and of its uncut part - Fig.1;

$S_i = d_0^2(\varepsilon - \sin\varepsilon)/8$;

$\varepsilon = 2\arccos(2b_s / d_0 - 1)$;

b_s - height of the incised part of the jet;

$b_s = R\sin\varphi_i - z_0 + d_0/2$;

c_1, c_2 - average values of the velocity at the inlet and outlet section;

φ_M - angle, determining the position of the application point M.

Substituting the values from equations 11-15 for the values in equation (10), yields the following expression for M_x:

$$M_x = \rho Q_i c_1 (z_0 - d_0/2 + b_s/2).[1 - \frac{c_2}{c_1}(\cos\beta_2 + \sin\beta_2 ctg\varphi_M)] \ . \tag{16}$$

For the value of the average velocity at the inlet section to be determined, it is necessary that the fluid-velocity profile of the free circular turbulent jet be calculated. It is apparent that the deflector operate at the initial section of the jet (Fig.2)- therefore, the following expression could be used for the border layer of the jet [3]:

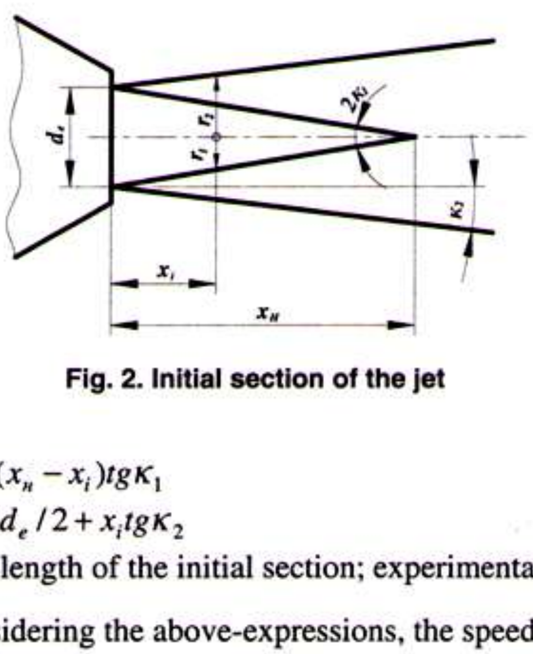

Fig. 2. Initial section of the jet

$$c = c_m(1 - 3\bar{r}^2 + 2\bar{r}^3) , \tag{17}$$

where:

c_m - speed/ velocity in the nucleus;

$c_m = k_c\sqrt{2gH}$;

k_c - speed / velocity coefficient; determined in accordance with the expression $k_c = f(H)$ [4];

$\bar{r} = r/\delta$ - thickness of the border layer of the stream; for the initial section $\delta = r_2 - r_1$;

$r_1 = (x_н - x_i)tg\kappa_1$

$r_2 = d_e/2 + x_i tg\kappa_2$

$x_н$ - length of the initial section; experimental-study value $x_н = 7d_0$.

Considering the above-expressions, the speed/ velocity c is determined as follows:

$$c = c_m[1 - 3(\frac{r - r_1}{r_2 - r_1})^2 + 2(\frac{r - r_1}{r_2 - r_1})^3] . \tag{17a}$$

Since it is necessary that the average value of the velocity at various positions of the deflector be determined, the following expression for estimating the flow average is used:

$$c_{1s} = \frac{\int_Q c\,dQ}{Q} = \frac{\int_s c^2 dS}{Q}. \qquad (18)$$

For the average velocity of the outlet section to be determined, the Bernouli's equation is being implemented :

$$c_2 = c_1\sqrt{1-\varsigma+2g\Delta z/c_1^2}\,, \qquad (19)$$

where:

ς - general coefficient of resistance;

Δz - difference in the height of the inlet and outlet sections; in the case of vertical-shaft (вертикален вал) turbines $\Delta z = 0$.

The hydraulic loss in the particular case is presented as a sum of the hydraulic loss of friction and change of velocity direction, and of the loss from collision at the deflector entry. Therefore, the general coefficient of resistance could be derived in the same fashion: $\varsigma = \varsigma_1 + \varsigma_2$.

The coefficient of resistance ς_1 could be determined by analogy to the coefficient of resistance at fluid current in a crank with rectangular section [5]:

$$\varsigma_1 = \{0.124 + 3.1[\frac{b_1}{2(R-b_1/2)}]^{3.5}\}\frac{2\theta}{\pi}, \qquad (20)$$

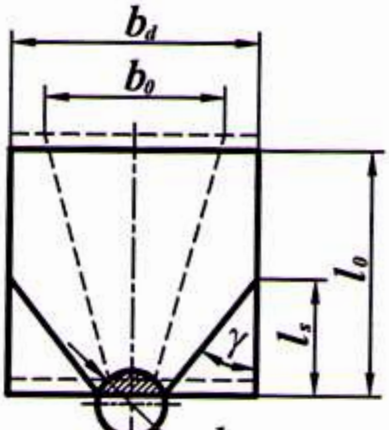

Fig. 3. Variation of the jet

where:

b_1 - thickness of the layer at the deflector outlet;

$$b_1 = \frac{Q_i}{c_2 b_0}, \qquad (21)$$

b_0 - stream width at the deflector exit (Fig.3);

θ - angle of the crank.

Experimental research shows that at the initial period of the deflector's incision $b_0 < b_d$ (b_d - deflector's width), while for all other positions $b_0 = b_d$. It is not difficult to establish that in the former case

$$b_0 = a_s + 2l_0 tg\gamma\,, \qquad (22)$$

where

$a_s = d_0 \sin\varepsilon$ (Fig. 1),

$l_0 = R\varphi_0$.

Tests confirmed that γ angle could be assumed $\gamma = \frac{\pi}{2}\varphi_i$. .

θ angle could be derived from the expression:

$$\theta = \varphi_0 - l_s / R , \tag{23}$$

$$l_s = \frac{b_d - a_s}{2} tg\varphi_i , \tag{24}$$

i.e. only the loss in the lower part of the deflector, in which it is filled up, are accounted for.

In first approximation, in accordance with the recommendations [1], the velocity c_2 is assumed to be $c_2 = 0.9c_1$.

Determining the coefficient of resistance ς_2 is accomplished through implementation of data from research done by LMZ and VIGM [1]. The following regression dependence is found in accordance with the mentioned data:

$$\varsigma_2 = \frac{\varphi_0}{\beta_1}(0.216 - 0.0426\overline{Q} + 0.1256\overline{Q}^2 - 0.1499\overline{Q}^3), \tag{25}$$

where $\overline{Q} = Q_i / Q_d$.

Thus determined, the values of ς_1 и ς_2 are used for calculation of the average value of the velocity at the outlet section in second approximation; afterwards the procedure is repeated (the obtained value is final).

The angle φ_M (fig.2) is determined in the following way:

$$tg\varphi_M = \frac{z_M}{x_M} . \tag{26}$$

The following equations are valid for the coordinates x_M and z_M:

$$z_M = z_0 - d_0 / 2 + b_s / 2 ,$$

$$x_M = \frac{R}{\sin\beta_2} - (z_M ctg\beta_2 + \frac{b_1}{2\sin\beta_2} .$$

After a substitution in equation (26) for the angle φ_M, the result is:

$$tg\varphi_M = \frac{z_M \sin\beta_2}{R - z_M \cos\beta_2 + b_1/2}. \tag{26a}$$

The friction force of water upon the stream-lined surfaces of the deflector could be determined through the tangential stresses τ and the stream-lined area S_c:

$$F_T = \tau\ S_c. \tag{27}$$

The tangential tensions are determined through the formula for longitudinally stream-lined wafer [7]:

$$\tau = c_f \rho \frac{c_1^2}{2}. \tag{28}$$

The coefficient of resistance c_f is calculated in accordance with the research of Nikuradse [7] by means of the following formula:

$$c_f = \frac{0.0267}{\mathrm{Re}^{0.139}}.$$

After the streamlined area is determined, the force F_T is to be obtained as follows:

$$F_T = \frac{6.675}{\mathrm{Re}^{0.139}} c_1^2 (a_s + b_0) l_0 \quad (\text{at } l_s > l_0), \tag{29a}$$

$$F_T = \frac{13.35}{\mathrm{Re}^{0.139}} c_1^2 [(a_s + b_0)\frac{l_s}{2} + (l_0 - l_s)(b_d + 2b_s) \quad (\text{at } l_s < l_0). \tag{29b}$$

The force F_T results in a momentum $M_T = F_T R$. The outcome of the numerical experiments shows that, in some cases, that momentum exerts significant influence upon the operation of the deflector.

5. NUMERICAL EXPERIMENTS

In accordance with the above-described algorithm has been developed a special software program DEFLEC, which affords the opportunity for developing optimal design of deflectors for Pelton water turbines. This program is also useful in determining certain geometric and kinematical characteristics of a deflector in various instances of its incision into the jet.

With the aid of the program have been calculated the so-called momentum characteristics $M_x = f(\alpha_i)$ of deflectors, for which there are data from experimental research in the specialized literature. Fig. 4 shows such a characteristic obtained in a model re-

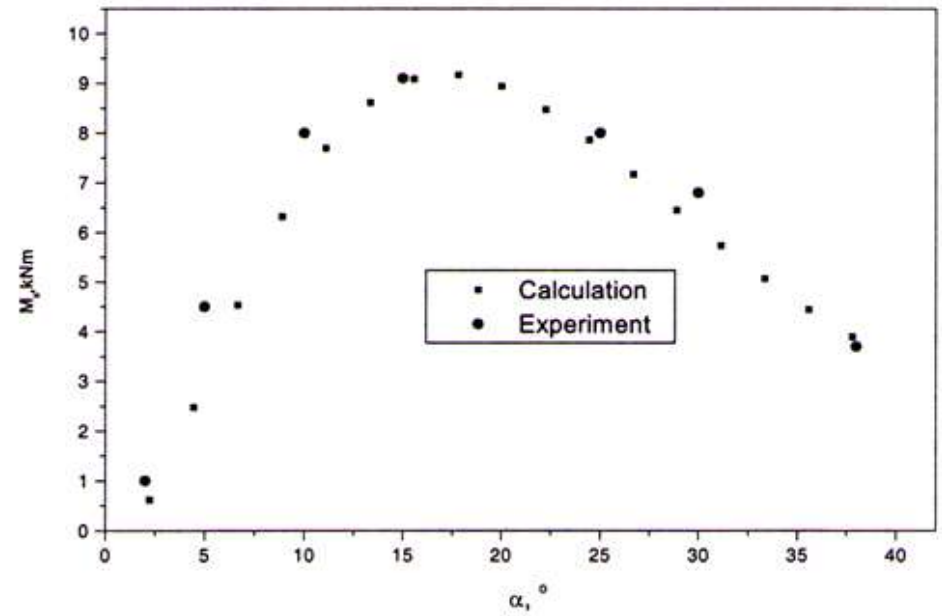

Fig. 4. Comparison of momentum characteristics (model turbine)

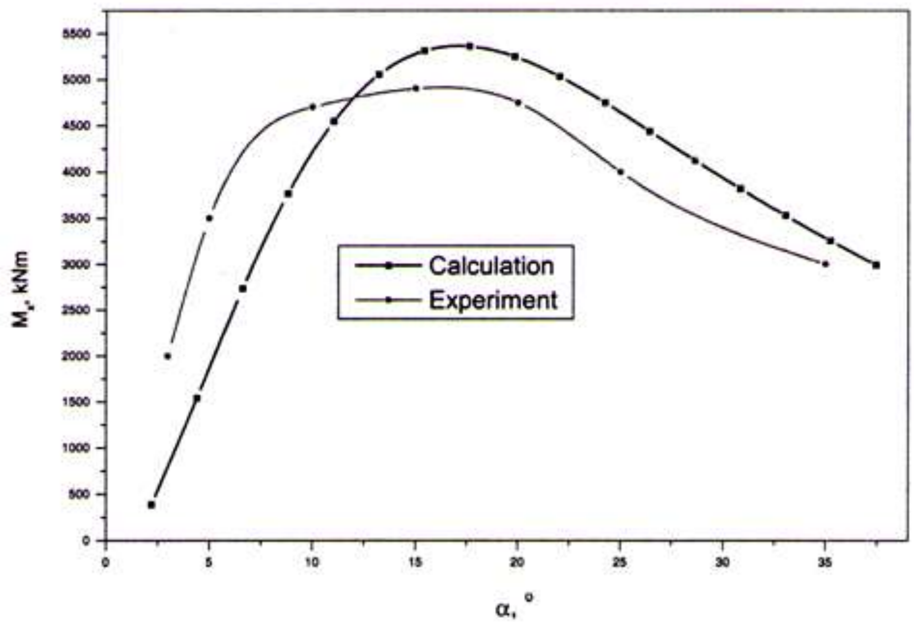

Fig. 5. Comparison of momentum characteristics (Tatevska HPS)

search of Pelton turbine at the laboratory of LMZ, Russia (H = 40 m, Q = 0.046 m^3/s). Fig. 5 demonstrates another one arrived at in a research held at the Tatevska Hydropower Station Russia (H = 568 m, Q = 11 m^3/s) [1]. Both figures — comprising experimental-research data — are compared with the data produced by calculation with the DEFLEC program.

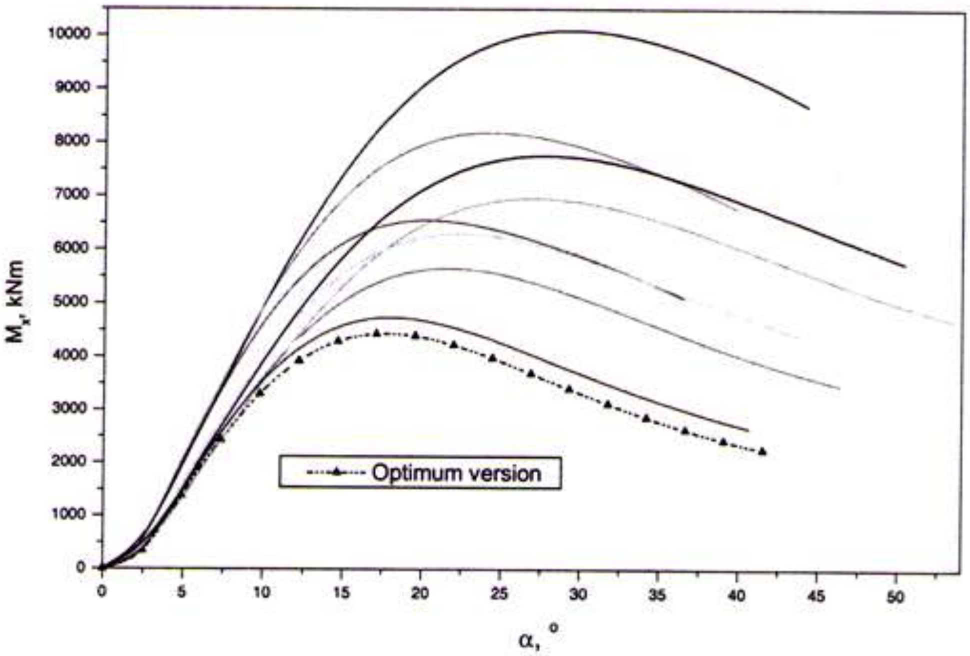

Fig. 6. Momentum characteristics of all calculated variants of deflectors

Fig.6 presents the momentum characteristics of all variants of deflectors, calculated with the aid of the DEFLEC program with data input from the Tatevska HPS, according to the points in the schedule of the numerical experiment. The characteristic of the optimum variant is also shown. It could be noted that the value of the maximum momentum (at $z_0 = -0.303m; x_0 = -0.035m$) is lower than the one of the operating turbine ($z_0 = -0.326m; x_0 = -0.032m$). This example properly demonstrates the qualities of the developed methodology and program for optimization of the momentum characteristics of the Pelton-turbine deflectors.

5. CONCLUSION

The developed methodology and calculation program afford an opportunity for accomplishing optimum design of deflectors for Pelton water turbines, especially in view of the torque, which is necessary for their operation control.

The comparison of the momentum characteristics, arrived at both through the methodology described at 3) and through experimental research, show satisfactory coincidence. The DEFLEC program can be implemented in research of deflectors in the process of modernization of HPS's with dated equipment. It can also be used as a module of a computer system for automated design of Pelton turbines.

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